IMAGE DENOISING ALGORITHM BASED ON TEMPLATE WAVELET COEFFICIENTS

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Abstract

Noise reduction and image edge enhancement are important problems in image processing. In the present work we propose an algorithm of noise reduction in the signals, which presents slowly varying functions between abrupt edges. Examples of such signals include satellite images of planets, radio images of atmospheric inhomogeneities and so on. The high frequency noise component is the main reason of signal and image defacement. However, use of the low frequency filtering procedure leads to edge smoothing and loss of small details. So, for improving the image it is necessary to reduce high frequencies in the space between the image edges of the image features while retaining them in the vicinity of the edges. Thus, it is necessary to analyze the signal local properties both in the frequency and in the time domain. Wavelet transformation technique is the best way to solve this problem. However, there are still problems concerning noise reduction in the vicinity of edges and suppressing parasitic oscillations, which arise when standard wavelet algorithms are applied. The proposed algorithm of image filtering with wavelet transform is based on the replacement of the original wavelet coefficients of the edges by template ones. The approach calls for the application of a specific analyzing wavelet. As the first step of application of this proposed technique, Earth's surface images obtained by side looking airborne radar have been processed.

1 Introduction

Noise reduction and image edge enhancement belong to typical problems in image processing. This paper deals with the problem of noise reduction in signals, which are slowly varying functions between vertical edges. Strings and columns of images obtained in experiments or observations are examples of such signals. For example, radar images of clouds have slowly varying reflectivity between sharp cloud edges. The high–frequency noise components produce the main distortions in the signals and images since the power

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of the high–frequency component of the signal is often less than the power of the low–frequency component. However, an application of the conventional low frequency filtration results in smoothing of edges and loosing of fine features. To keep the vertical edges, the high frequencies should be removed between edges and retained in the vicinities of edges. For this purpose, one can use a representation of the signal, which allows analyzing local properties of signals both in the time and in the frequency domain. Windowed Fourier transform is an example of such a representation, but it gives a spectrum of the signal segment which occurs in the window. A disadvantage of this transform is related to the fixed time resolution determined by the window width. Wavelet transforms provide a more accurate time–frequency representation of the signals. The continuous wavelet transform (CWT) of a signal $f(t)$ is introduced as [Daubechies, 1992; Combes et al., 1989]

$$W\left(a,b\right)=\int\limits_{-\infty}^{\infty} f(t)\psi^{*}_{a,b}(t)dt, \tag{1}$$

$$\psi_{a,b}(t)=\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), a\in R^{+}, b\in R, \tag{2}$$

where $\psi(t)$ is a mother wavelet. The symbol * stands for complex conjugation. The wavelet spectrum $W(a,b)$ is a function of two variables: a scale parameter $a>0$ (the analog of frequency) and a translation parameter b (location of the time window). The wavelet transform gives a local spectrum of the signal in the vicinity of b, similar to the windowed Fourier transform. However, the time and frequency resolutions of the wavelet transform are dependent: a high–frequency wavelet has a narrow time window, and vice versa, a low–frequency wavelet has a wide time window. It also should be noted that there are a lot of different wavelets and one can choose an appropriate wavelet depending on the features of the problem to be solved. In practice, the discrete wavelet transform (DWT) is usually applied [Daubechies, 1992; Shensa, 1992; Rioul and Duhamel, 1992], when samples of the wavelet spectrum, called wavelet coefficients, are calculated. Samples of the wavelet spectrum are taken in the nodes of a grid on the plane (a,b),

$$d_j[k]=W\left(a_j,b_{j,k}\right); j,k\in Z. \tag{3}$$

In this paper we use the DWT which produces undecimated wavelet coefficients:

$$a_j=2^j; b_k=k; j,k\in Z. \tag{4}$$

Wavelet coefficients with the same octave number j and various k–values form an octave. The inverse discrete wavelet transform was introduced in order to reconstruct the original signal from its wavelet coefficients. An algorithm of the direct undecimated DWT is given by

$$c_{j+1}[k]=\sum_{n} h^{*}[n]c_j[k+2^j n], \tag{5}$$

$$d_{j+1}[k] = \sum_n g^*[n] c_j[k + 2^j n], \tag{6}$$

and allows to calculate recursively the coefficients $d_j[k], j = 1, 2, ..., J$, and $c_J[k]$ from the scaling coefficients $c_0[k]$. An inverse undecimated DWT algorithm is given by

$$c_j[k] = \frac{1}{2}\sum_n \tilde{h}[n] c_{j+1}[k - 2^j n] + \frac{1}{2}\sum_n \tilde{g}[n] d_{j+1}[k - 2^j n], \tag{7}$$

and yields coefficients $c_0[k]$ from the coefficients $d_j[k], j = 1, 2, ..., J$, and $c_J[k]$.

The presence of sharp variations of a signal or of edges in an image leads to the appearance of peaks in each octave of the wavelet coefficients. Such peaks are called edge peaks. The position of these peaks indicates the position of edges. Similar peaks arise also due to the presence of noise which naturally complicates the analysis. The main idea of so far used algorithms for filtration of the wavelet coefficients consists of the following: The wavelet coefficients that describe the edge peaks are preserved, whereas the rest of the coefficients are eliminated. This leads to a reduction of the noise influence. The difference between various algorithms is mainly due to the rule, which is used to determine the correct coefficients describing the edge peaks. The wavelet threshold filtering is the most popular algorithm for this purpose. According to this algorithm, wavelet coefficients, which exceed some threshold value, are considered as correct ones. The threshold value is chosen either heuristically or with the help of special algorithms [Donoho, 1995; Donoho and Johnstone, 1994; Xu et al., 1994]. This approach has two disadvantages. At first, deleting the damping tail of edge peaks leads to the appearance of parasitic oscillations in the reconstructed signal. At second, coefficients satisfying the threshold condition are not eliminated so that noise in the vicinity of edges is not filtered. In this paper, a novel approach to the filtration of wavelet coefficients is proposed in order to get rid of these drawbacks. The main idea of the approach consists in the application of template wavelet coefficients, which are used for replacing the noisy wavelet coefficients. It has been found that a specific type of analyzing wavelet should be used in order to realize the proposed technique. It has also been found that the first derivative of the cubic B–spline is the particularly appropriate choice for the analyzing wavelet.

2 The algorithm of template wavelet coefficients

The shape of the edge peak is determined by the shape of the wavelet and by the shape of the edge. The vertical edge is the simplest one and a signal with a single vertical edge is the Heaviside function

$$\theta(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}, \tag{8}$$

The undecimated wavelet coefficients of the Heaviside function, called below as template wavelet coefficients, in accordance with (1), (2), are given by

$$d_j^\theta[k] = \frac{1}{\sqrt{2^j}} \int_{-k}^{\infty} \psi^* \left(\frac{t}{2^j}\right) dt, \tag{9}$$

These coefficients describe the edge peaks, which correspond to an ordinary discontinuity of unit height of any function. Let us consider a signal, which is a slowly changing function between vertical edges. Such signal can be represented as

$$f(t) = s(t) + \sum_m H_m \theta(t - T_m), \tag{10}$$

where $s(t)$ is a slowly varying function, T_m and H_m are the coordinates and heights of the vertical edges. The wavelet coefficients of such a signal are given by

$$d_j[k] = \frac{1}{\sqrt{2^j}} \int_{-\infty}^{\infty} s(t)\psi^* \left(\frac{t-k}{2^j}\right) dt + \frac{1}{\sqrt{2^j}} \sum_m H_m \int_{T_m}^{\infty} \psi^* \left(\frac{t-k}{2^j}\right) dt. \tag{11}$$

We will assume that the slowly varying function $s(t)$ coincides with the approximation of the function $f(t)$ with the resolution of J, and therefore is wholly described by the scaling coefficients $c_J[k]$ (see the theory of the multiresolution analysis in Mallat [1989] or Daubechies [1992]). Then their contribution to the wavelet coefficients with $j < J$ equals zero. These wavelet coefficients, called below as clean wavelet coefficients, are completely determined by the locations and heights of the edges T_m and H_m by the template wavelet coefficients $d_j^\theta[k]$,

$$d_j^c[k] = \sum_m H_m d_j^\theta[k - T_m]. \tag{12}$$

The locations of edges T_m can be found as the positions of extremes on the wavelet coefficients with sufficiently large octave number $d_J[k]$, and the heights of edges can be calculated from the extreme wavelet coefficients by the expression

$$H_m = \frac{d_J[T_m]}{d_J^\theta[0]}. \tag{13}$$

Some threshold may be used to distinguish the edge peaks from those produced by noise. This threshold defines the minimal height of edges. The basic idea of this approach is the following: From the analysis of wavelet coefficients with large octave number J one can find positions T_m and heights H_m of edges for a particular signal. After that, by using the template coefficients (9), one can calculate the clean wavelet coefficients $d_j^c[k]$ with $j < J$. These coefficients are used to replace real noisy wavelet coefficients $d_j[k]$ for all j. Supplementing the clean wavelet coefficient by retained scaling coefficients $c_J[k]$ and carrying out the inverse DWT, we have obtained the filtered signal. As a result, the noise will be reduced to a large extent, and spurious parasitic oscillations do not arise. Moreover, it should be noted that slow time variations of the signal are modeled correctly since they are mainly described by the scaling coefficients. Thus the algorithm of template wavelet coefficients allows the noise reduction not only between edges, but also in their vicinity, and parasitic oscillations do not arise. Moreover, the algorithm allows reconstructing the vertical edges from smoothed, corrupted by the noise edges.

3 Choice of the wavelet

Let us consider the wavelet in the form of a derivative of an even square integrable function $G(t)$,

$$\psi(t) = dG(t)/dt, \tag{14}$$

$$G(-t) = G(t). \tag{15}$$

Then the expression (6) for the template wavelet coefficients is reduced to

$$d_j^{\theta}[k] = -\sqrt{2^j}G\left(\frac{k}{2^j}\right). \tag{16}$$

To find the locations and heights of edges with a high accuracy, the edge peaks should be well localized and have a single extreme, pointing the edge location. It means that the function $G(t)$ should have a bell–like shape.

For example, $G(t)$ can be taken in the form of Gaussian. Then the wavelet will be the first derivative of the Gaussian. However, the cubic B–spline $\phi_3(t)$ is the more appropriate choice for the function $G(t)$.

$$\phi_3(t) = \begin{cases} \frac{1}{6}(t+2)^3, & -2 \le t \le -1 \\ -\frac{1}{6}(3t^3+6t^2-4), & -1 \le t \le 0 \\ \frac{1}{6}(3t^3-6t^2+4), & 0 \le t \le 1 \\ -\frac{1}{6}(t-2)^3, & 1 \le t \le 2 \\ 0, & |t| > 2 \end{cases}, \tag{17}$$

The Fourier transform of the cubic B-spline is

$$\hat{\phi}_3(\omega) = \frac{1}{\sqrt{2\pi}}\left(\frac{\sin\omega/2}{\omega/2}\right)^4. \tag{18}$$

Table 1: Multiresolution analysis filters for derived algorithm.

n	$g[n]$	$h[n]$	$\tilde{g}[n]$	$\tilde{h}[n]$	$h_3[n]$
-3	–	–	-1/16	–	–
-2	–	–	-1/16	–	$\sqrt{2}/16$
-1	–	$-\sqrt{2}/4$	1/2	$\sqrt{2}/8$	$\sqrt{2}/4$
0	-1	$3\sqrt{2}/4$	1/2	$3\sqrt{2}/8$	$3\sqrt{2}/8$
1	1	$3\sqrt{2}/4$	1/16	$3\sqrt{2}/8$	$\sqrt{2}/4$

It turns out that for the chosen wavelet exact pyramidal direct and inverse DWT algorithms with the finite–length filters can be built. The formulas of the pyramidal DWT can be obtained from the formulas of the undecimated biorthogonal DWT (5),(6),(7) substituting $d_{j+1}[k]$ by $d_j[k]$, $j = 0, 1, 2, ..., J-1$, with the filters given in Table 1. We have obtained this pyramidal algorithm from the well known biorthogonal multiresolution analysis (MRA) with the quadratic B–spline as the scaling function [Daubechies, 1992] by using the expression that relates the chosen wavelet $\psi_B = d\phi_3/dt$ to the wavelet ψ_2 from the above mentioned biorthogonal MRA,

$$\hat{\psi}_2\,(2\omega) = -\frac{1}{2}\exp\left(-i\omega\right)(2 + cos\omega)\hat{\psi}_B\,(2\omega) \tag{19}$$

For the chosen wavelet, the clean wavelet coefficients (12) are given by

$$d_j^c[k] = -\sqrt{2^j}\sum_m H_m\phi_3\left(\frac{k-T_m}{2^j}\right) \tag{20}$$

We have found that the clean wavelet coefficients can be calculated in a fast way. Namely, since the cubic B–spline is the scaling function of biorthogonal MRA [Daubechies, 1992], the clean wavelet coefficients can be calculated with the equation (5) with filter $h_3[k]$ (see Table 1). In fact, it is sufficient to calculate the clean wavelet coefficients $d_0^c[k]$ for $j = 0$ directly from (20), and then take them as the coefficients $c_0[k]$ for (5). By carrying out the calculation in this way, one can easily see that the required clean wavelet coefficients are equal to

$$d_j^c[k] = 2^j c_j[k], j = 0, 1, 2, ..., J-1. \tag{21}$$

The choice of the cubic B–spline as a function $G(t)$ allowed us to build simple, effective and exact direct and inverse pyramidal DWT algorithms with the finite length filters, and organized the computation of the clean wavelet coefficients in a fast way. Another choice requires more complex algorithms than the proposed pyramidal. Therefore, the choice of the cubic B–spline can be considered as the particularly appropriate.

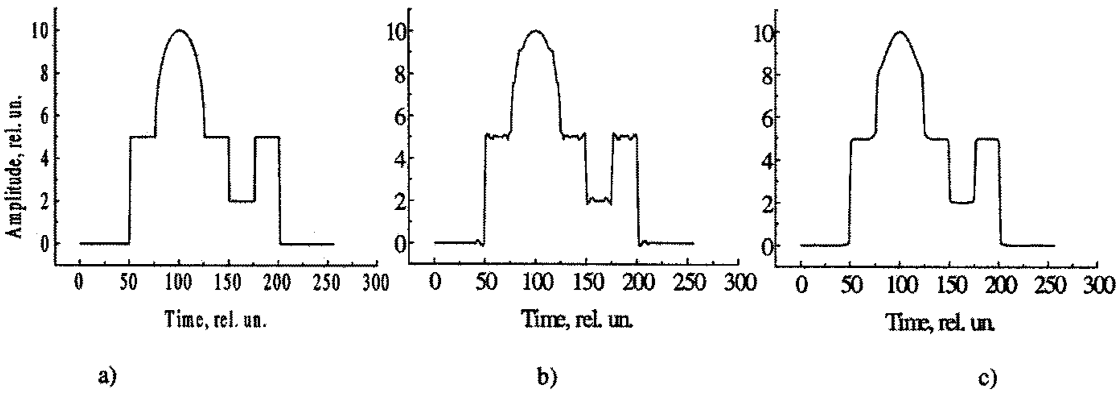

Figure 1: Noiseless test signal processing: (a) Noiseless test signal, (b) Test signal after threshold filtering, (c) Test signal after the proposed filtering technique.

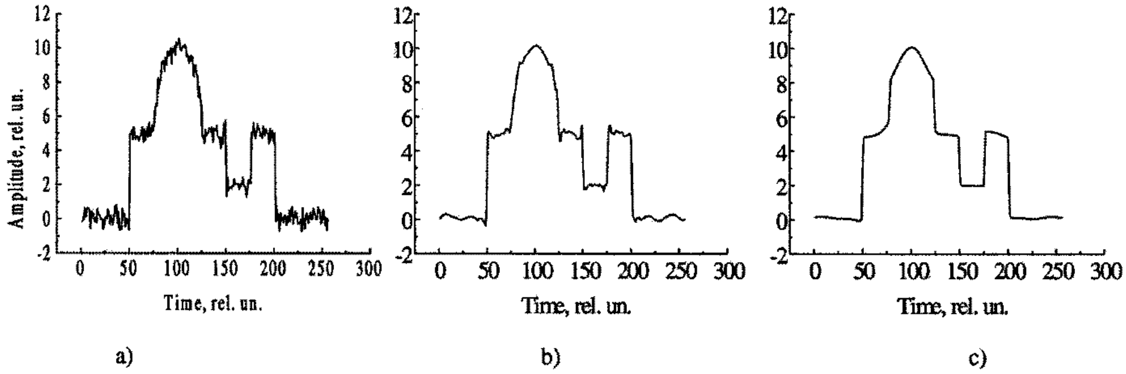

Figure 2: Noisy test signal filtering: (a) Noisy test signal, (b) Test signal after threshold filtering, (c) Test signal after the proposed filtering technique.

4 Conclusion

Thus the application of the proposed noise reduction technique allows us to avoid the excitation of parasitic oscillations. The result of applying the proposed algorithm to a noiseless test signal (Figure 1a) is shown in Figure 1c. The result of applying the threshold algorithm to the same noiseless test signal is shown in Figure 1b. As one can see, parasitic oscillations in the vicinity of edges are totally absent if the proposed algorithm is used. This is in contrast to the threshold filtering technique. Moreover, the proposed technique allows an effective noise reduction of the signal not only between the edges but also in the vicinity of the edges as well. The noisy test signal, shown in Figure 2a, was processed by using the threshold technique (Figure 2b) and the proposed algorithm (Figure 2c). By comparing both figures, one can see that the noise reduction is rather good when the proposed algorithm is used. We have found that the clean wavelet coefficients can be calculated in a fast way. The choice of derivative of the cubic B–spline as a wavelet allowed us to build simple, effective and exact direct and inverse pyramidal DWT algorithms with the finite length filters, and organized the computation of the clean wavelet coefficients in a fast way. Another choice requires more complex algorithms than the proposed pyramidal.

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